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PREFACE

Preface

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Complementarity and duality are closely related, multi-disciplinary topics that pervade all natural phenomena, and form the basis for solving many underlying nonconvex or global optimization problems that arise in science and engineering. During the last 40 years, much research has been devoted to the development of mathematical modeling, theory, and computational methods in this arena. The field has now matured in convex systems, especially in linear programming, engineering mechanics and design, mathematical physics, economics, optimization, and control.

In mathematical programming and analysis, the subject of complementarity is closely related to constrained optimization, variational inequality, and fixed point theory. Through the classical Lagrangian duality, the KKT conditions of constrained optimization problems lead to corresponding complementarity problems. The primal-dual schema has continued to evolve for linear and convex mathematical programming during the past 20 years. However, for nonconvex systems, it is well-known that the KKT conditions are only necessary under certain regularity conditions for global optimality. Moreover, the underlying nonlinear complementarity problems are fundamentally difficult due to the non-monotonicity of the nonlinear operators, and also, many problems in global optimization are NP-hard. The well-developed Fenchel–Moreau–Rockafellar duality theory will produce a so-called duality gap between the primal problem and its Lagrangian dual. Therefore, how to formulate perfect dual problems (with a zero duality gap) is a challenging task in global optimization and nonconvex analysis. Extensions of the classical Lagrangian duality and the primal-dual schema to nonconvex systems are ongoing research endeavors. In parallel, many new algorithmic advances in global optimization are enabling the solution of heretofore

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open, difficult engineering design and process control problems to global optimality for the very first time in the literature.

In mathematical physics and mechanics, the concept of complementarity leads itself to perfect duality (without a duality gap), which is due to certain conservation laws that govern the system. The original idea of this one-to-one complementary-duality is from the traditional Chinese Yin-Yang philosophy. Bohr realized its value in quantum mechanics. His complementarity theory and philosophy laid a foundation on which the field of modern physics was developed. In classical mechanics, each energy function is linked, via the Legendre transformation, with a complementary energy through which the Lagrangian and the Hamiltonian can be formulated. For convex static systems, the Lagrangian is a saddle function(al) which leads to a one-to-one (mono) duality between the potential variational principle and the so-called complementary variational principle. This is the well-known saddle min–max duality in convex optimization. For dynamic systems, the convex Hamiltonian leads to an interesting bi-duality between the total action and the dual action, i.e., the double-min and double-max dualities. The classical Legendre transformation also plays a key role in nonconvex mechanics. Actually, the well-known generalized Hellinger–Reissner complementary energy principle can be viewed as the first nonconvex Lagrangian duality theory. Based on this principle and the complementary-duality, a unified canonical duality framework in mathematical physics has been proposed in the work by Gao and Strang. This framework reveals an intrinsic duality in nonconvex systems and through which a so-called canonical duality theory has gradually developed, first in nonconvex mechanics and then in global optimization. This canonical duality theory is composed mainly of a canonical dual transformation and a triality theory. The canonical duality theory can be used to formulate perfect dual problems with zero duality gaps, while the triality theory can be used to identify both global and local optimality conditions. This potentially powerful theory can be used to solve many nonconvex/nonsmooth problems in nonlinear analysis, engineering mechanics, and global optimization. However, due to the existing gap among these fields, the fundamental idea of the complementarity-duality in physics and the potential importance of the canonical duality theory have not been fully understood.

In view of the importance of complementarity-duality theory and methods in global optimization, and in order to bridge the ever-increasing gap between global optimization and engineering science, the First International Conference on Complementarity, Duality, and Global Optimization (CDGO) was held at Virginia Tech, Blacksburg, August 15–17, 2005, under the sponsorship of the National Science Foundation. This conference brought together more than 100 world-class researchers from interdisciplinary fields of Industrial Engineering, Operations Research, Pure and Applied Math, Engineering Mechanics, Electrical Engineering, Psychology, Management Science, Civil Engineering, and Computational Science. The conference spawned some new trends in optimization and engineering science, and has stimulated young faculty and students to venture into this rich domain of research.

This special issue of the *Journal of Global Optimization* contains 16 papers from selected lectures presented at the Conference. These papers deal with fundamental theory, algorithms, and applications of complementarity and duality in multidisciplinary fields of global optimization, including nonlinear programming, nonsmooth and nonconvex variational/optimization problems in mathematical physics and engineering mechanics, vector variational inequalities, optimal design and control of engineering structures, as well as NP-hard problems in computational science.

The completion of this special issue would not have been possible without the assistance of many of our colleagues. We wish to express our sincere appreciation to all those who helped. In particular, our special thanks to Professor P. M. Pardalos for inviting us to edit this special issue and for his enthusiastic encouragement. We are also deeply grateful to selected anonymous referees who provided prompt and insightful reviews for all the submissions. Their constructive comments have greatly contributed to the quality of the volume. Finally, we thank Alice Clawson and Sandy Dalton for their able administrative and editorial assistance.